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ABSTRACTS

Conference on Noise and Chaos in
Nonlinear Dynamical Systems

Institute for Scientific Interchange
Villa Gualino
Viale Settimo Severo 65
10133 Turin, Italy

7 - 11 March 1989

Sponsors:

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Phase Sensitive Studies of Periodic and Chaotic Dynamics and Noise in Lorenz-Like Systems

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Heterodyne measurements of Lorenz-like behavior of an optically pumped far-infrared ammonia laser operating at 157 nm reveals a new world of symmetric and antisymmetric solutions, homoclinic and heteroclinic forms of chaos, and important phase dynamics for further study. This laser can behave almost indistinguishably from the five-equation model for the single mode laser with a homogeneously broadened two-level medium which is isomorphic to the complex Lorenz model. Qualitative features found in the models such as instability thresholds, types of pulsations, inverse period doublings, period-doubling and intermittent chaos, and periodic windows within chaos are also found in the experiments.

Heterodyne detection of the complex electric field amplitude of the laser (corresponding to the x-variable of the Lorenz model) reveals that the laser can behave as either the laser-Lorenz model or as the three-level, optically-pumped-laser model. These are distinguished by the presence of heteroclinic ('class Lorenz-type spirals') chaos and homoclinic chaos. We also demonstrate that the symmetric and antisymmetric periodic solutions for a resonantly-tuned laser (which is isomorphic to the Lorenz model with real variables) can be observed and studied. This will permit exploration of alternative routes to chaos including the symmetry breaking and symmetry restoring sequences of periodic pulsations that have been studied by Coulet and Tresser.

Out of resonance we find important phase dynamics that accompany the more frequently periodic pulsations. During the peak of the laser pulse the optical frequency is pulled closer to resonance with the medium and then a dramatic 'slip-page' of phase occurs between pulses.

The complex Lorenz equations were demonstrated by Sarker and Carmichael to be extremely sensitive to noise which induces phase diffusion of the attractor of the real Lorenz equations and which thereby results in an increase in the dimension of attractors reconstructed from the real part of either of the two complex variables. A similar phase diffusion occurs in the noise-free complex equations with small detuning. We compare the theoretical predictions for the noisy-resonant laser and the noise-free detuned laser and discuss the possibility that these different kinds of behavior can be resolved in further experimental measurements.

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38 3128 (1988)

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Chaos and Noise in a Laser With Saturable Absorber

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The influence of internal and externally added noise on the operation of an infrared laser with an intracavity saturable absorber has been investigated. In particular, the effect of the noise on the unstable and chaotic regimes of the laser and on the transition between these regimes has been examined. The relation between the detection of homoclinic chaos in the laser with saturable absorber and the noise level in the laser system will be discussed.

**Noise-Sustained Structure and Convective Instabilities
in Amplitude and Phase Equations**

Helmut R. Brand

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We^{1, 2}, show that above but close to the onset of many instabilities with a finite value for the group velocity (as is the case for traveling waves) many systems are convectively unstable, that is perturbations grow as one is moving downstream, but decay when observed at a fixed location. Under convectively unstable conditions localized noise or perturbations applied upstream can generate periodic or turbulent patterns further downstream leading to noise-sustained structures, which vanish upon removal of the noise. The applicability of this concept to a number of patterns (including 'football' and 'confined' states) observed recently near the onset of thermal convection in binary fluid mixtures is critically examined.

¹ R. J. Deissler and H. R. Brand, Physical Letters A, in print

² H. R. Brand and R. J. Deissler, submitted for publication

Strange Attractor for Cyclists

Predrag Cvitanovic

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The same deterministic dynamical system can be presented to us in many obscurely related guises: as a phase-space trajectory, as a time-delay plot, as a map such as a Poincare section of a flow. The fundamental question of the theory of dynamical systems is: what are the invariants of the system, ie. the numbers that can be extracted from the system no matter how we represent it?

The simplest such invariant properties are topological; a fixed point must be a fixed point in any good representation, a trajectory which exactly returns to the initial point (a cycle) must do so in any good representation. Furthermore, we shall show that also the invariant metric content of a deterministic strange set can be encoded in terms of its periodic orbits in a convergent manner, by associating curvatures to the primitive cycles. Recently developed methods for analysis of experimental low dimensional strange attractors make feasible accurate extraction of their periodic orbits, a measurement which, we propose, should supplant the current practice of extracting only the global averages (such as the Lyapunov exponents and generalized dimensions).

Low Dimensional Behavior in the Complex Ginzburg-Landau Equation

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The complex Ginzburg-Landau equation in one spatial dimension with periodic boundary conditions is studied from the viewpoint of effective low dimensional behavior by three distinct methods. Linear stability analysis of a class of exact solutions establishes lower bounds on the dimension of the universal attractor and the Fourier spanning dimension, defined here as the number of Fourier modes required to span the universal attractor. We utilize concepts from the theory of inertial manifolds to determine rigorous upper bounds on the Fourier spanning dimension, which also establishes the finite dimensionality of the universal attractor. Upper bounds on the dimension of the attractor itself are obtained by bounding (or, for some parameter values, computing exactly) the Lyapunov dimension and invoking a recent theorem that asserts that the Lyapunov dimension, defined by the Kaplan-Yorke formula, is an upper bound on the Hausdorff dimension. This study of low dimensionality in the complex Ginzburg-Landau equation allows for an examination of the current techniques used in the rigorous investigation of finite dimensional behavior. Contact is made with some recent results for fluid turbulence models, and we discuss some unexplored directions in the area of low dimensional behavior in the complex Ginzburg-Landau equation.

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Random Walk and Trapping in Hierarchical Systems

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Random Walk processes on hierarchically organized structures can model the anomalously enhanced pair separation of test particles in developing turbulence. Depending upon the scaling parameters of the model, regimes of diffusional transport (power-law, exponential, divergent) separated by dynamical phase transitions can be investigated by an analytical characterization of the eigenvalue set of the underlying master equation. Inclusion of the possibility of trapping or decay of the wandering particles produces several new effects as diffusion limited decay and decay controlled spreading.

**The Optical Parametric Oscillator:
A Model System for Quantum Noise Reduction in Optics**

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Université Pierre et Marie Curie, Paris, France

Parametric interaction between pump, signal and idler waves inside an optical cavity, although one of the simplest non-linear processes in optics, displays a large variety of working regimes: linear amplification below the oscillation threshold, generation of c. w. fields at the signal and idler frequencies, self-pulsing regime, chaotic behavior. In some circumstances, bistability between two of these regimes can also occur. Furthermore, the quantum fluctuations of the emitted fields exhibit non-classical effects, especially when the Optical Parameter Oscillator (OPO) is close to a transition threshold between the previously mentioned regimes. This leads to large quantum noise reduction effects, either for the quadrature components of the emitted fields, their intensities or the difference between the intensities of the signal and idler waves. We will review the different situations which can be encountered and describe experimental observations of large quantum noise reduction in the OPO.

Testing Approximate Theories of Colored Noise

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In recent years the modeling of noise in physical systems by white noise has been superseded by modelling with exponentially correlated, colored noise. This change reflects a move towards greater physical realism in modeling. From a purely theoretical point of view, it produces a more difficult mathematical problem because of the non-Markovian nature of colored noise. For weak noise, approximate Markov treatments have been proposed. Alternatively, embedding of the problem in higher dimension produces a Markovian picture. Either way, mathematical tractability is in general lost. Numerical simulation techniques have recently undergone dramatic improvement which yields highly accurate and quickly obtained results. These techniques will be presented and discussed with relation to the theoretical approaches. Applications to mean first passage times and the effects of colored noise will be highlighted.

Spatial Chaos and Selfsimilarity in Developed Turbulence

Siegfried Grossmann

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High Reynolds number flow exhibits spatio-temporal chaos. The attractor has a finite Hausdorff-Lyapunov-dimension although the underlying equations of motion (Navier-Stokes) are partial differential equations. Also based on the NS-equations one can derive the dependance of the eddy energy $\langle\langle v^2(r)/2 \rangle\rangle$ on their size r , for the viscous range (small r) as well as for the universal range (larger r), including the transition range. The key for the understanding is the Lagrangian time correlation decay which can also be found from the NS-equations. Applications to passive scalar distribution and to turbulent diffusive transport are briefly touched. The state of the art is a mean field type treatment of the NS-equations. Agreement with the data is promising.

Dissipative Tunneling

P. Hänggi

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The quantum analogue of Kramers reaction theory is derived using a unique many-body approach that is valid at all temperatures. At high temperatures, the standard, separable many-body quantum transition state theory is shown to be equivalent to the Kramers reaction theory with tunneling corrections included. The theory is applied to tunneling in chemical reactions such as adiabatic electron transfer in polar media and tunneling of heavy molecules in heme proteins. There exists a characteristic temperature T_0 below which tunneling events dominate over activated hopping processes. This crossover temperature to tunneling controlled reactions sensitively depends on the form of the vibrational spectra of the surrounding medium. Moreover, we present new results for the quantum rate in presence of external periodic fields (ie. quantum resonance activation).

Cavity Q.E.D.: Probing the vacuum Field Noise in a Confined Space

S. Haroche

Ecole Normale Supérieure and Yale University

The spectral field distribution of vacuum field fluctuations is drastically modified in a closed space surrounded by metallic boundaries. An atom placed in such a confined space has its radiative properties radically altered. Spontaneous emission can be either enhanced or inhibited. The atomic energy levels are shifted. Studying the atom's properties in the modified environment is a way to probe the characteristics of the altered vacuum.

This research on atom-vacuum interaction belongs to a new field of atomic physics and quantum optics called Cavity Quantum Electrodynamics (Cavity QED). It has also led to the creation of microscopic masers operating with a single atom and a few photons in the cavity. The field radiated by these new quantum devices exhibit interesting non-classical properties. Moreover, the operation of a micromaser provides a very simple illustration of the Quantum Theory of measurement applied to a quantum mechanical oscillator. A review of recently performed Cavity QED experiments will be presented.

Colored Noise in Bistate Systems

Peter Jung and Peter Hänggi

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We consider the overdamped motion in the archetype bistable potential $V(x) = Bx^4 - Ax^2$ driven by exponentially correlated Gaussian noise with the correlation time τ . The smallest nonvanishing eigenvalue of the underlying two dimensional Fokker-Planck equation is calculated numerically for small-to-moderate-to-large correlation times, $\tau=0$ to 2, at very weak noise intensities, $D=0.03$ to 0.1. This eigenvalue has the physical meaning of an escape rate. Numerous approximations of the τ -dependence of this escape rate, often with conflicting or misleading results, have been put forth leading the casual reader to a state of confusion. We find three different regimes in the correlation time dependence of the escape rate: the regimes of small correlation times, moderate correlation times and of very large correlation times. Approximations in these different regimes are compared with our precise numerical results, for which the error is $<0.1\%$.

Chemical Waves, Coupled-Map Lattices and Cellular Automata

R. Kapral

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Wave propagation and pattern formation in chemically reacting systems will be discussed. The emphasis of the discussion will be on the construction of discrete dynamical models of such systems. Two types of model will be considered: coupled-map lattices, where both space and time are taken to be discrete variables but the state space is continuous, and lattice gas cellular automata, where space, time and particle velocities are discrete. The discussion will focus on the nature of the phenomena observed, the extent to which discrete models can capture the observed behavior, the comparison between the coupled-map lattice descriptions and the more microscopic lattice gas cellular automata and the role of fluctuations on the pattern formation and evolution processes.

Phase-Dependent Interaction of Squeezed Radiation With Atoms

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A. Ekert

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In this paper we review the current state of progress in research on squeezed light. We discuss the basic theory of squeezing the nature of phase-dependent quantum noise in optical fields. We review various nonlinear sources of squeezed light and their properties. We then discuss the properties of squeezed light in interaction with atoms, showing how the phase dependence of the field fluctuations transfers itself to the atoms through dipole decay and through frequency shifts, so that the basic radiative corrections to atomic states become themselves "squeezed".

¹ On leave from Istituto di Fisica dell'Universita, Palermo, Italy

Quantum Theory of Measurement

Willis E. Lamb

University of Arizona, Tucson, Arizona, USA

An up-to-date summary of the author's views on the Quantum Theory of Measurement will be given. Most of his past publications in this area are listed below. A computer simulation of a realistic measurement process, based on the new model described in references 6 and 7, will be given in graphical form.

1. "An Operational Interpretation of Nonrelativistic Quantum Mechanics", *Physics Today* 22, 23-28 (1969).
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Pattern Formation in Diffusion-Limited Reactions

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Bruce J. West

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Diffusion-limited reactions in low-dimensional systems exhibit behavior very different from classical kinetics. This behavior has been observed in numerical simulations and also in actual experiments for the reactions $A + A \rightarrow \text{products}$ and $A + B \rightarrow \text{products}$. The differences from classical behavior are made evident in the form of the global rate law for these reactions and also in the spatial distribution of the reactions. In particular, the spatial distributions are nonhomogeneous: the reaction-diffusion dynamics leads to pattern formation in the concentrations of chemical species on a macroscopic scale. One must distinguish the behavior of these systems when the reaction is of the "big bang" type (ie. particle creation occurs only at the origin of time) and when it is in a steady state reaction in which reactants are continually supplied: the critical dimension below which the behavior is nonclassical depends on the specific reaction and also on these experimental conditions. We shall present a variety of results for such systems. We shall also consider the relation between the mesoscopic reaction-diffusion equations introduced at a phenomenological level and the underlying microscopic dynamics.

Swept Parameter Dynamics in the Presence of Noise

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C. Vanden Broeck

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Given a deterministic nonlinear dynamical system which displays a steady bifurcation point from a trivial solution, we analyze the competing effects (i) of sweeping the control parameter in time and (ii) of the presence of noise. Our aim is to locate the new position of the average bifurcation point.

The noise affects both the initial condition and the time evolution of the system during the parameter sweep. Thus a statistical average has to be performed over each complete trajectory.

For arbitrary sweep rates and arbitrary noise correlation times this problem can be reduced exactly to an implicit equation involving only elementary quadratures. In the limit of small sweep rates, we derive explicit analytical results for the limiting cases of small and large correlation times.

Stochastic Resonance in Bistable Systems

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The relaxation properties of a stochastic bistable system perturbed by a periodic low-frequency forcing term is investigated by means of analogue simulation. The so-called stochastic resonance phenomenon is revealed under diverse experiment conditions. Its dependence on the relevant process parameters, namely the amplitude and frequency of the perturbation, the external noise statistics and the damping constant is described theoretically.

**Spatio-Temporal Instabilities in Nonlinear Optical Systems:
Recent Theoretical and Experimental Progress**

L. Narducci and J. R. Tredicce

Department of Physics, Drexel University, Philadelphia, Pennsylvania, USA

**Chaotic Dynamics and Markovian Coarse-Graining in
Nonlinear Dynamical Systems**

G. Nicolis

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A general procedure for casting deterministic chaos into a discrete Markov process is derived. The method is illustrated on simple examples such as the logistic map. The implications of the results in the foundations of statistical mechanics and in the problem of predictability are discussed.

**Non-Markovian Dissipative Processes from a
Generalized Quantum Langevin Equation Viewpoint**

R. F. O'Connell

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Baton Rouge, Louisiana, USA

We will consider the following problems:

- (a) the exact solution for the free energy of a charged quantum oscillator in a black-body radiation field and its relevance to experiments on the dynamic (AC) Stark shifts of high Rydberg atomic states.¹
- (b) non-Markovian effects on dissipative quantum tunneling.²
- (c) non-linear transport for the many-body system.³
- (d) the independent oscillator model of a very general heat bath and, concomitantly, the serious defects associated with linear coupling models such as the oft-used Ullersma mode.⁴

We show that the generalized quantum Langevin equation⁵ provides an all-embracing framework for the analysis of all these problems.

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- 4. P. Ullersma, *Physica* 32, 27 (1966).
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Algebraic Structure of the Dressed Jaynes-Cummings Model

Mario Rasetti

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Resorting to a one-fermion realization of the Pauli operators, the Hamiltonian of the Jaynes-Cummings model is recognized as an element of the superalgebra $u(1|1)$, which plays the role of dynamical algebra. The extension of this notion to the superalgebra $osp(2|2)$ allows adding to the JC hamiltonian both virtual and real two-photon processes. The exact diagonalization of the resulting new hamiltonian ("dressed JC") can be performed by algebraic methods. Interesting features, such as squeezing, can hence be derived, both in the case when the coupling constants of the new model — which are elements of a subfield of Banach-Grassmann numbers -- are assumed to be nilpotent and when are identified with c-numbers. Finally, the algebraic approach to the related Fokker-Planck equation, in terms of supercoherent states, is thoroughly discussed.

Quantum Treatment of Dispersive Optical Bistability

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First the model of Drummond and Walls (DW) is introduced by which dispersive optical bistability is treated in a fully quantum mechanical way. The basic equation of the DW model is an equation for the density operator of the light mode inside the cavity. By using the Glauber-Sudarshan P-function or the Q-function the equation of motion for the density operator is transformed into a pseudo Fokker-Planck equation (FPE), i.e., a FPE where the diffusion matrix is not positive definite or semidefinite. For the Wigner-function the diffusion matrix is positive definite but additional third-order derivatives occur. It is shown that the pseudo FPE and the equation for the Wigner-function can be solved in terms of matrix continued fractions. Explicit results are given for the stationary Q- and Wigner-function as well as for the lowest nonzero eigenvalue. For vanishing thermal fluctuations this eigenvalue determines the tunneling rate between the two 'bistable' states.

Signals, Noise and Stochastic Resonance: Does the Weather Affect Lasers?

Rajarshi Roy, B. McNamera and K. Wisenfeld

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The detection of signals in the presence of noise has long motivated research in the fields of radio, microwave and optical physics. Many ingenious techniques have been invented and fascinating physical phenomena discovered in the effort to extract signals from a noisy background. We will address issues related to the super-regenerative amplification and detection of optical signals and to the phenomenon of stochastic resonance. The latter was used by Benzi et. al. to explain to 10^5 year periodicity in the earth's ice ages.¹ We present evidence for this phenomenon in the operation of a bistable ring laser. It is shown that the signal to noise ratio in the output of the ring laser with a periodically modulated asymmetry between the clockwise and counterclockwise waves actually increases with the injection of noise into the system.

1. R. Benzi, G. Parisi, A. Sutera and A. Vulpiana, *Tellus* 34, 10 (1982).

Time Scales of Systems Driven by Colored Noise

J. M. Sancho

Universidad de Barcelona, Barcelona, Spain

The scaling laws for the characteristic times of a system under the influence of colored noise are evaluated using very simple techniques. Explicit results for the transient behavior of unstable, marginal and metastable initial states are presented. A comparison with digital and numerical simulations is made.

**Transition Phenomena in Multi-Dimensional Systems —
Models of Evolution**

W. Ebeling, A. Engel, L. Schimansky-Geiser

Sektion Physik, Humbolt-Universität zu Berlin, Berlin, DDR

Three types of transition in stochastic systems are investigated:

- i) The activation process in one- and two-dimensional bistable systems driven by exponentially correlated noise is studied. The stationary distribution processes the skew asymmetry recently observed by several authors numerically and experimentally.
- ii) Harmonic and chaotic noise is introduced by coupling a stochastic oscillator to one- and two-dimensional bistable systems. Skew stationary distributions are obtained and tendencies for mean transition times were given. The Unified Colored Noise Approximation is applied to derive simpler kinetic equations for the probability distribution.
- iii) Results for the activation process in multistable multi-dimensional systems are presented for the purpose to model evolutionary processes in phenotype spaces and optimization in high dimensional search spaces. The Fisher-Eigen model is investigated and some general properties of evolution are discussed.

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**The Degenerate Quantum Beat Laser:
Lasing Without Inversion and Inversion Without Lasing**

M. O. Scully

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**Center for Advanced Studies, Department of Physics and Astronomy,
University of New Mexico, Albuquerque, New Mexico**

In the correlated spontaneous-emission laser, we find gain without population inversion and optical pumping to excited states without radiative decay.

**Field Pattern Bistability in a Laser
Oscillating in Two Frequency-Locked Transverse Modes**

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The characteristics of a laser which oscillates simultaneously in nearly degenerate TEM_{01} and TEM_{10} Gauss-Hermite resonator modes are investigated. An analytical model is formulated which is based on coupled Maxwell-Bloch-type laser equations for the two oscillating modes. The model predicts that below a critical value for the difference of the empty-cavity mode eigenfrequencies, both modes lock on a common oscillation frequency. Here, the two transverse modes maintain a common optical phase difference close to $+\pi/2$ or $-\pi/2$, thereby forming a TEM^*_{01} hybrid mode pattern. Although the *intensity* patterns that correspond to the two possible phase-locking angles are identical, the *field* patterns are mirror images of each other. Laser operation is bistable with respect to the two field patterns; switching between the stable states is possible by injection of suitable optical fields. The theoretical results are illustrated by experiments using a Helium-Neon laser.

Suppressing Quantum Noise

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Squeezed light has been used to perform interferometric and quantum nondestruction measurements beyond standard quantum limits. A review of noise suppression techniques employing squeezed light, with emphasis on experiments, will be given.

From Chaos to Symmetry

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Not so long ago we understood that chaos may appear from the order if dynamical systems are nonlinear. But, now it is clear much more: dynamical chaos conserves some kind of order. By the help of the chaos new symmetries may be eliminated, and onset of chaos is a result of competition between interactions of different symmetries.

Stochastic webs are new universal phenomena which realizes quasi symmetries from chaotic motion. We consider different systems with stochastic webs: particle in fields, 3-D hydrodynamic flows and patterns formation.

Description of the diffusion on multifractals is included as typical property of the dynamical systems with webs. Generalized Levy fly process is considered.

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POSTERS

**Intensity Fluctuations of Amplified Spontaneous Emission:
Statistics, Correlation Functions, Tests for Chaotic Behavior**

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Noise Reduction and Instabilities in Semiconductor Lasers With Optical Feedback From a High-Q External Cavity

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The amplitude and frequency noise in a semiconductor laser have been reduced using optical feedback from a high finesse external cavity resulting in a laser linewidth of a few tens of kilohertz. However, the optical feedback also enhances the relaxation oscillations driving the laser unstable if the feedback is too strong. Experimental measurements are compared with results of theoretical modeling. We find that the optical feedback serves to integrate and reduce the fluctuations arising from spontaneous emission and from carrier number shot noise and $1/f$ noise. The strong noise reduction occurs only within the bandwidth of the external resonator. From our results we have developed a simple intuitive picture of the effect of the external resonator in control-circuit language and we provide information of ways to optimize the coupling of the laser and external cavity from maximum stability and minimum linewidth.

**Systematic Errors in the Determination of Entropies and Dimensions of Chaos
in Data with Limited Precision and Additive Noise**

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Stochastic Dynamics of Nonlinear Dynamical Systems

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Co-ordinated experimental, numerical and theoretical studies of the influence of random external fluctuations (noise) of both the regular and chaotic dynamics of carefully chosen archetypal nonlinear oscillators are in progress. Particular attention is being paid to the physical ROBUSTNESS (engineering integrity) of the various attractors (harmonic, sub harmonic, chaotic) against noise inducing transitions across basin boundaries. Preliminary results will be presented for escape from an under-damped, potential well, a generic problem that is of universal importance in science and engineering.

**Mean First Passage Times in Bistable Systems Driven by Colored Noise:
New and Exact Results in the Correlation Time Limit**

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We show that the Local Linearization Theory can be used to define unambiguously the Mean First Passage time in systems where standard theoretical approaches would lead to ill defined equilibrium distributions in some regions of the support. LLT predictions are tested against numerical simulations in an archetypal bistable system of connected parabolas and the agreement with data is shown to be very good even in parameter regions where standard techniques would yield either a negative diffusion coefficients or a negative damping. Furthermore, we prove that any residual disagreement in the regions of large theoretical approach: the proof is based on the explicit computation of such prefactors for a quartic potential, where the corrected theory and the computer simulations virtually coincide.

Thermal Activation in Bistable Systems Under External Periodic Forces

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Considered is the motion of a Brownian particle in a bistable potential exposed to an external periodic field. Our analysis is based on a systematic Fokker-Planck description of the non-stationary stochastic process. Besides general characteristic, such as the non-mixing property, we present full numerical solutions for probability distributions, the modulation induced rate enhancement and the dynamical susceptibilities.

**Stochastic Relaxations in Bistable Nonequilibrium Systems
Driven by Colored Noise**

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Universal Normal-Form Description of Squeezing in Two-Photon Processes

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So far the nonlinear interactions most efficient in producing significant levels of squeezing seem to be those based on a two-photon interaction of a cavity mode. We show that when the damping constant of the cavity mode is much larger than all other relaxation states, the system develops an unstable domain and the squeezing in an appropriate quadrature component of the output field tends to become perfect as one approaches the instability threshold, and can be described in terms of a simple universal normal-form. This connection between instabilities thresholds and strong squeezing was first noted by Collet and Walls [M. J. Collet and D. F. Walls, *Physical Review A* 32, 2887 (1985)], who found perfect squeezing at critical points (both steady bifurcations, turning points and Hopf bifurcations), and has been observed in the recent intracavity second harmonic generation experiment of Kimble and collaborators (H. J. Kimble, S. Pereira and Min Xiao, Technical Digest of the XVI International Quantum Electronics Conference, paper WB-1, p. 466).

We work out the examples of two-photon optical bistability, both from a numerical and an analytical viewpoint, degenerate four-wave mixing and second harmonic generation. Moreover, we show that in the bad-cavity limit the two-photon laser with injected signal exhibits a significant level of squeezing in the phase quadrature component: this reduction of noise in an active device is quite an interesting feature.

Uniform Asymptotic Expansions in Dynamical Systems Driven by Colored Noise

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We construct asymptotic expansions of the (quasi) stationary probability density function and of the mean first-passage time over a potential barrier, for bistable systems driven by weak wideband Gaussian colored noise, when the intensity ϵ and the correlation of time τ of the noise are both small. Previous analyses have led to a variety of often contradictory results and to considerable confusion, which stem from the fact that the problem depends on two small parameters. This results in different expansions, with different ranges of validity, depending on the relative magnitudes of ϵ and τ . In contrast, we derive expansions that are uniformly valid throughout the entire parameter range of interest. In addition, we identify restrictions on the ranges of validity, in terms of the total power output ϵ/τ of the noise, of previously derived expansions. We show that only if the power output ϵ/τ becomes infinite can previously derived expansions be valid. Our results, when specialized to this case, reduce to expansions previously derived. Outside the restricted range, i.e., for finite or vanishingly small power outputs, our expansions contain terms which are new, and which may in fact dominate previously computed terms. In contrast to the use of one-dimensional diffusional approximations previously employed, our approach is based on the exact two-dimensional Fokker-Planck equation. Singular perturbation techniques, previously developed by the authors, are employed to systematically derive the asymptotic expansions.

Relaxation Near Noise-Induced Transitions

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The increase in system correlation time T often observed in the vicinity of noise-induced transitions is considered taking, as an example, the quadratic model recently investigated by Leung (Physical Review 1988). Analogue and digital experiments on this model failed to reveal any sign of critical slowing down, or of the divergence of T predicted to occur near the survival/extinction noise-induced transition, thereby motivating fresh theoretical approaches. A new, exact, analytic calculation of T as a function of noise intensity is presented and shown to be in excellent agreement with experiment. The origins of the earlier (misleading) theoretical prediction are discussed.

References:

H. K. Leung, Physical Review A 37, 1341 (1988)

Colored Noise: The Point of View of Path Integrals

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A stationary distribution for Langevin equations driven by colored noise is obtained, in the weak noise limit, from the configuration space Lagrangian-like function. The derivation makes no explicit use of Markovian, Fokker-Planck or small correlation time approximations. Markovian approximations based on the Lagrangian, which do not involve truncated expansions, are also discussed.

Random Walk in Disordered Chains: Some New Results

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A variety of models of random walk in static and dynamically disordered chains are studied. For time dependent disorder, models with white site-independent (global) and white site-dependent (local) disorder are considered and exact results are given in terms of an effective random walk in a nondisordered medium. For local disorder we find a slower diffusion while for global disorder the continuous limit of the effective random walk is not a diffusion process.

The case of static disorder is analyzed through a novel diagrammatic expansion based on Terwiel's cumulants. For the strong disorder B and C models [S. Alexander et al., Review of Modern Physics 53, 175 (1981)], we prove that the Effective Medium Approximation (EMA) gives the exact result in the calculation of the diffusion coefficient $D(\omega)$ up to order smaller than ω . The leading correction to the EMA's result for $D(\omega)$ is calculated.

**Dynamic (AC) Stark Shifts for Atoms Immersed
in Nonclassical States of the Radiation Field**

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Harmonic Noise

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A new stochastic process which generalizes the Ornstein-Uhlenbeck process is introduced. Harmonic noise is an oscillator driven by white noise. The spectrum possesses a maximum at the oscillator frequency. The effect of harmonic noise on bistable oscillators is investigated by several methods. Stationary probability distributions and mean first passage times were calculated approximately.

A Bifurcation in Squeezed State Physics: But How?

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The area of overlap principle together with the concept of interference in phase space serves as a guide to the phase distribution of a Gaussian cigar squeezed state and illuminates a striking new feature: a **bifurcation** forced by the control parameter built out of the displacement and the squeeze.

Fractal Dimensions and Bifurcation Structure of NH_3 -Lasers

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We present comparisons between observed dynamics of the $80\text{ }\mu\text{m}$ NH_3 -laser and predictions of the Lorenz equations. Agreement is shown in the structure of the two-dimensional bifurcation diagram (pump and detuning as control parameters).

Detailed comparison of the correlation dimensions of the attractors, reconstructed from measured and calculated intensity, as well as comparisons of the spectra of generalized dimensions give further clear evidence of the high degree of correspondence of the NH_3 -laser dynamics with the one of the (complex) Lorenz model.

**First Passage Times in the Presence of Noise
in the Laser With Saturable Absorber**

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